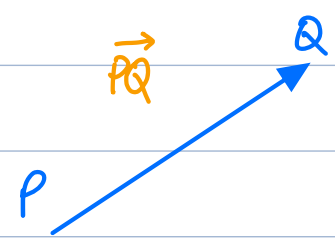
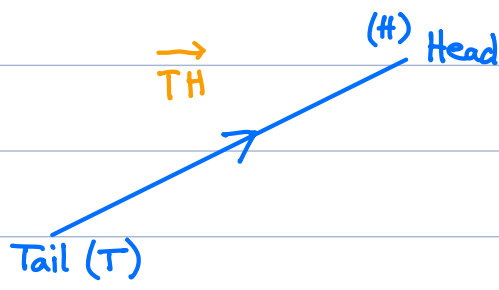
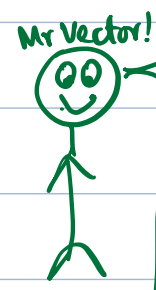


VECTORS (5-6 Marks)

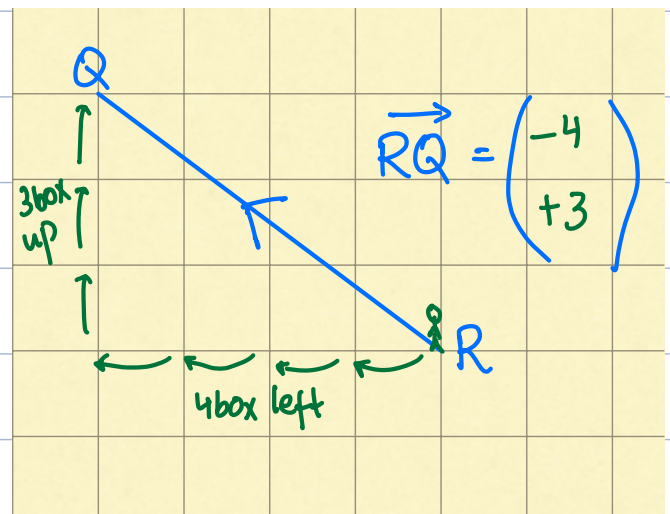
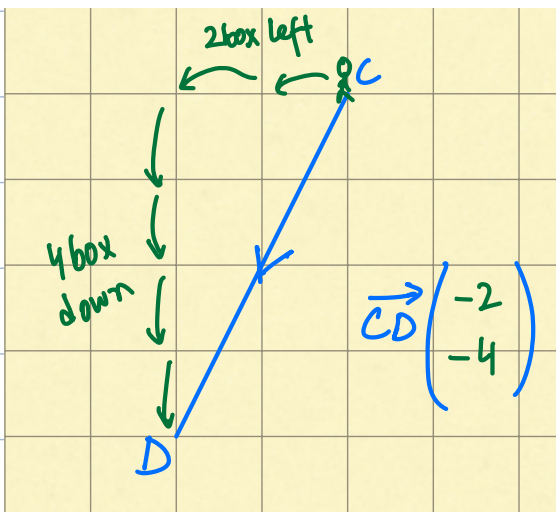
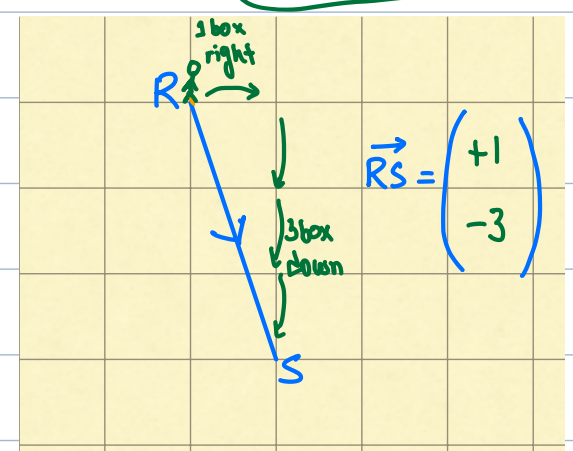
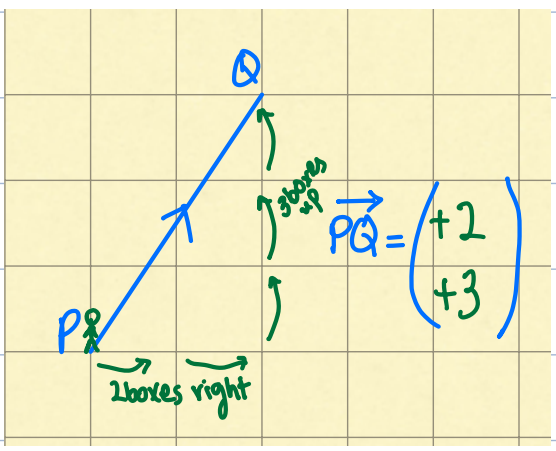


COLUMN VECTOR

$\begin{pmatrix} x \\ y \end{pmatrix}$
 $\begin{matrix} \oplus & \ominus \\ \text{right} & / \text{left} \\ \oplus & \ominus \\ \text{up} & / \text{down} \end{matrix}$



I will start from tail and go to head.
 I can only move in 4 directions
 up/down/left/right
 I cannot move slant



MAGNITUDE (LENGTH) OF VECTOR:

If you have a vector $\vec{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$

Symbol: |name of vector|

THEN, MAGNITUDE (LENGTH) IS :

$$|\vec{AB}| = \sqrt{x^2 + y^2}$$

Q: $\vec{AB} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$, Find $|\vec{AB}|$

$$\begin{aligned} |\vec{AB}| &= \sqrt{(3)^2 + (-4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \end{aligned}$$

$$|\vec{AB}| = 5$$

Note: When we put values in any formula, use BRACKETS.

Q: $\vec{AB} = \begin{pmatrix} 6 \\ q \end{pmatrix}$, Given that $|\vec{AB}| = 10$

Find values of q .

$$|\vec{AB}| = \sqrt{(6)^2 + (q)^2}$$

$$(10)^2 = (\sqrt{36 + q^2})^2$$

$$100 = 36 + q^2$$

$$64 = q^2$$

$$q^2 = 64$$

$$q = \pm \sqrt{64}$$

Cautious: $\sqrt{b^2 + q^2}$
 $\sqrt{b^2} + \sqrt{q^2}$

You cannot open power separately in ADD/SUB

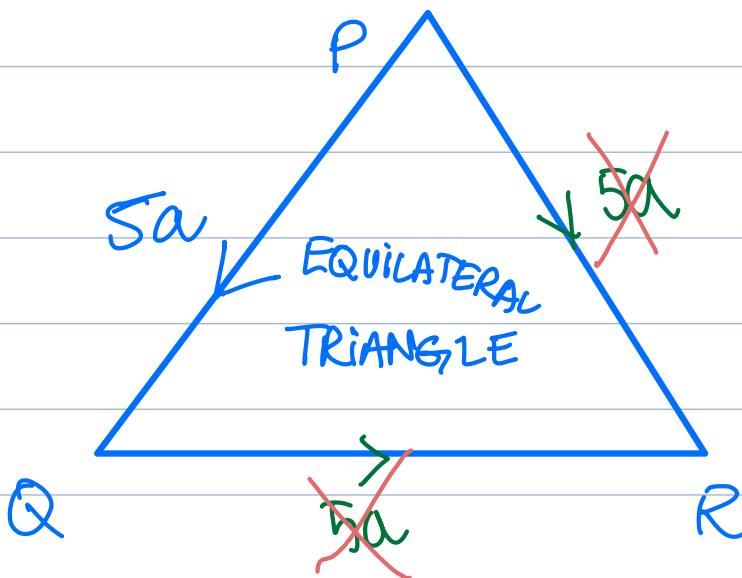
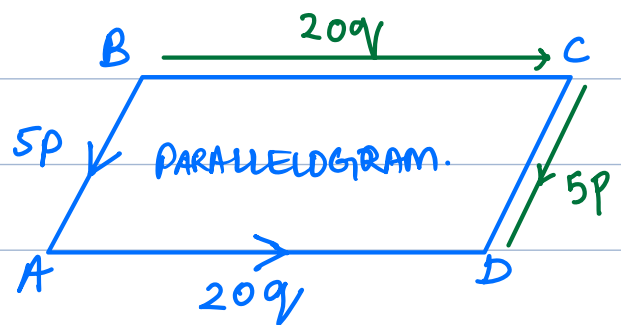
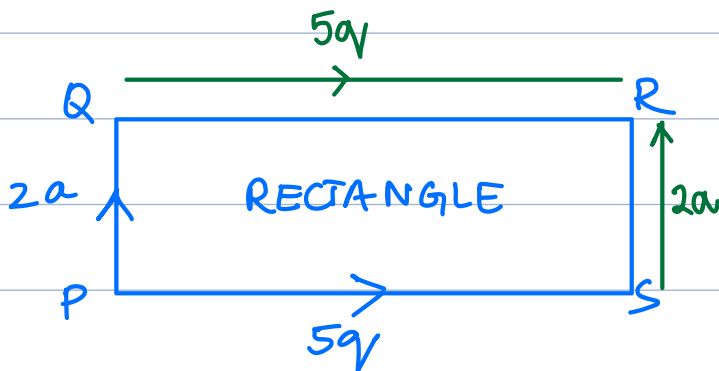
Whenever we take square root to cancel square, we introduce $\pm$ sign on side with square root.

$$q = \pm 8$$

most imp $\rightarrow$ **EQUAL VECTORS**

MUST SATISFY TWO CONDITIONS:-

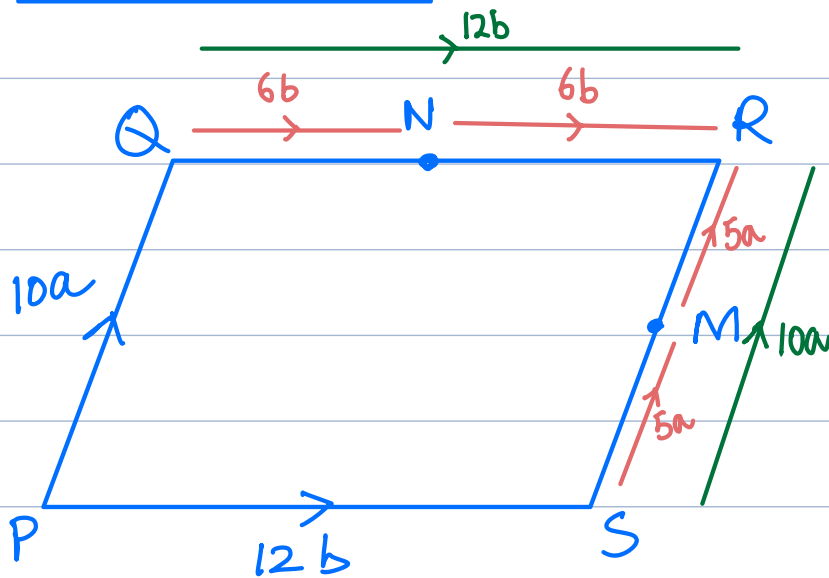
- 1) EQUAL LENGTH.
- 2) PARALLEL TO EACH OTHER.



There are no equal vectors here because to be equal the lines must be PARALLEL.

BASIC MODE

The $+/-$ signs rule for U/D/L/R are not applied here.



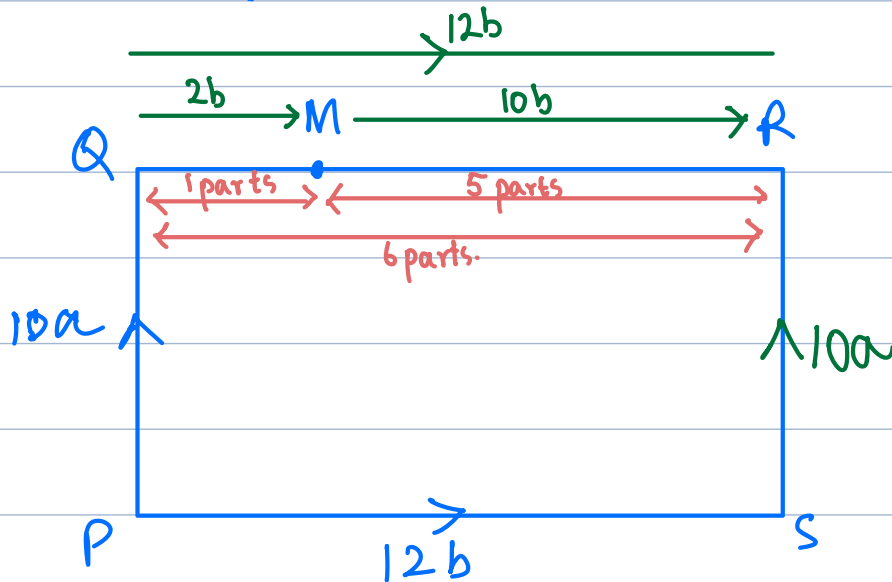
PQRS is a parallelogram.
M and N are midpoints.

Find:

- (a) $\vec{PR} = +10a + 12b$
 $+12b + 10a$
- (b) $\vec{QS} = +12b - 10a$
 $-10a + 12b$
- (c) $\vec{PN} = +10a + 6b$
- (d) $\vec{PM} = +12b + 5a$
- (e) $\vec{NS} = +6b - 10a$
- (f) $\vec{MN} = +5a - 6b$

ADVANCED MODE

(RATIO METHOD)



PQRS is a RECTANGLE.

$$QM = \frac{1}{5} MR$$

Find:

- $\vec{PR} = 12b + 10a$
- $\vec{QS} = 12b - 10a$
- $\vec{QM} = 2b$
- $\vec{PM} = 10a + 2b$
- $\vec{MS} = 10b - 10a$
- $\vec{RM} = -10b$

$$QM = \frac{1}{5} MR$$

Ratio Method:-

STEPS: 1) Correct form.

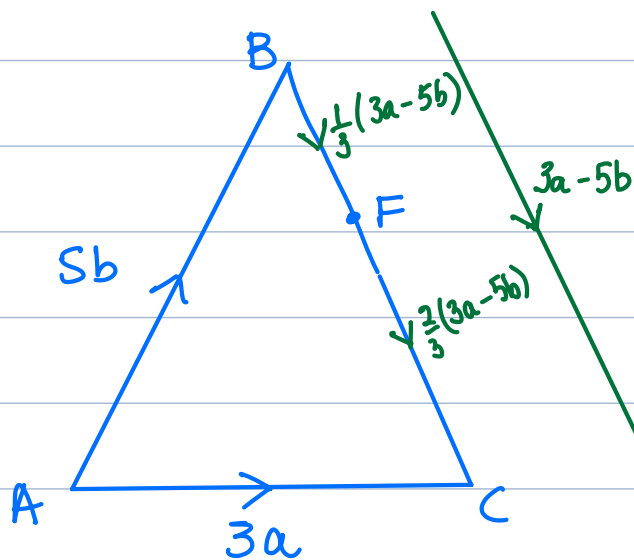
$$\frac{QM}{MR} = \frac{1}{5}$$

5 parts (above 1), 5 parts (below 5)

2) Mark parts on the diagram

3)		PARTS	ACTUAL	
(GIVEN)	$\vec{QR}$	6	12b	$6x = 12b \times 1$
(FIND)	$\vec{QM}$	1	x	$x = \frac{12b}{6}$
				$x = 2b$

Q:



Given that F is such that
 $BF = \frac{1}{3} BC$

Note for Shortcut in Ratio Method:

IF ONE OF THE VECTORS IN RATIO ARE KNOWN, JUST PUT THAT VALUE IN THE RATIO. NO NEED TO APPLY FULL THREE STEP RATIO METHOD

(i) Find $\vec{BC} = -5b + 3a = 3a - 5b$

EXAM TIP: It is better to write +ve terms before -ve terms.
 It helps us in workings of next parts.

(ii) Find $\vec{BF}$

$$\vec{BF} = \frac{1}{3} \vec{BC}$$

$$\vec{BF} = \frac{1}{3} (3a - 5b)$$

(iii) Find $\vec{AF}$.

$$3 \times 1 \cdot 5b + \frac{1}{3} (3a - 5b)$$

$$\frac{15b + 3a - 5b}{3}$$

$$\frac{10b + 3a}{3}$$

WORKING RULES:

1) EQUAL VECTORS

2) RATIO METHOD

(Always check for shortcut)

(Shortcut: If you know value of one of vectors in ratios, do not apply full ratio method).

Super Imp 3) Label each vector that you find on diagram. (Don't draw new lines)

4) Whenever you have fractions, take LCM.

In a difficult question

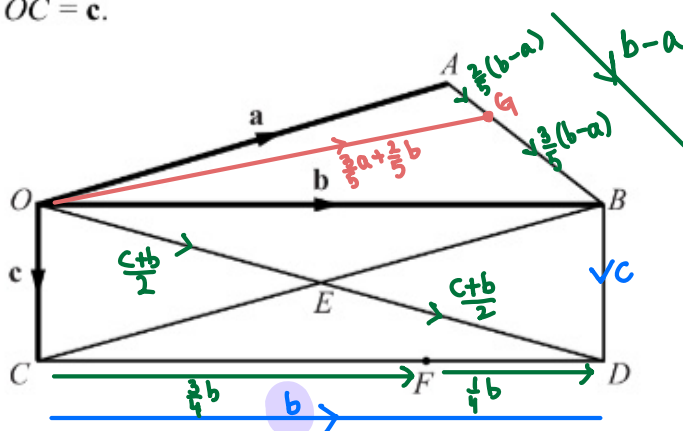
- Fraction $\rightarrow$ LCM
- Label everything.

- 8 OAB is a triangle and $OBDC$ is a rectangle where OD and BC intersect at E .
 F is the point on CD such that $CF = \frac{3}{4} CD$.

$\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$ and $\vec{OC} = \mathbf{c}$.

$$CF = \frac{3}{4} CD$$

$$CF = \frac{3}{4} b$$



- (a) Express, as simply as possible, in terms of one or more of the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$,

(i) $\vec{AB} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$

Answer $\mathbf{b} - \mathbf{a}$ [1]

(ii) $\vec{OE} = \frac{\vec{OB} + \vec{OC}}{2} = \frac{\mathbf{b} + \mathbf{c}}{2}$

Answer $\frac{\mathbf{b} + \mathbf{c}}{2}$ [1]

(iii) $\vec{EF} = \frac{2(\mathbf{b} + \mathbf{c})}{2} - \frac{1}{4}\mathbf{b}$
 $= \frac{2\mathbf{c} + 2\mathbf{b} - 1\mathbf{b}}{4}$
 $= \frac{2\mathbf{c} + \mathbf{b}}{4}$

Answer $\frac{2\mathbf{c} + \mathbf{b}}{4}$ [2]

- (b) G is the point on AB such that $\vec{OG} = \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$.

- (i) Express $\vec{AG}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
 Give your answer as simply as possible.

$$\vec{AG} = \frac{5}{5}\vec{OG} - \frac{5}{5}\vec{OA} = \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b} - \mathbf{a} = \frac{-5\mathbf{a} + 3\mathbf{a} + 2\mathbf{b}}{5} = \frac{2\mathbf{b} - 2\mathbf{a}}{5} = \frac{2(\mathbf{b} - \mathbf{a})}{5}$$

Answer [1]

- (ii) Find $AG : GB$.

$$\frac{2(\mathbf{b} - \mathbf{a})}{5} \div \frac{3(\mathbf{b} - \mathbf{a})}{5} = \frac{2(\mathbf{b} - \mathbf{a}) \times 5}{3(\mathbf{b} - \mathbf{a}) \times 5} = \frac{2}{3} = \boxed{2:3}$$

Answer 2 : 3 [1]

- (iii) Express $\vec{FG}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.
 Give your answer as simply as possible.

$$\frac{1}{5}\mathbf{b} - \frac{2}{5}\mathbf{c} - \frac{3}{5}(\mathbf{b} - \mathbf{a})$$

$$\frac{5\mathbf{b} - 20\mathbf{c} - 12(\mathbf{b} - \mathbf{a})}{20} = \frac{5\mathbf{b} - 20\mathbf{c} - 12\mathbf{b} + 12\mathbf{a}}{20} = \frac{12\mathbf{a} - 7\mathbf{b} - 20\mathbf{c}}{20}$$

Answer $\frac{12\mathbf{a} - 7\mathbf{b} - 20\mathbf{c}}{20}$ [2]

POSITION VECTORS

NAME STARTS WITH "O" O = origin (0,0)

$\vec{OP}$ = Position vector of P

$\vec{OA}$ = Position vector of A

[POSITION VECTOR
OF A POINT] = [COORDINATES
OF THAT POINT]

$\vec{OA} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ means A(3,5)

C(7, 5) means $\vec{OC} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$

WHENEVER YOU FEEL THAT
THERE IS A QUESTION MIXING

VECTORS and **COORDINATE GEOMETRY**

IT IS ACTUALLY
POSITION VECTOR.

$\vec{AB}$ has two points $A \rightarrow \vec{OA}$
 $B \rightarrow \vec{OB}$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{PQ} = \vec{OQ} - \vec{OP}$$

$$\vec{CX} = \vec{OX} - \vec{OC}$$

EXAM QUESTION

Q: $A(1, 3)$ and $\vec{AB} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$

Find coordinates of B .

$$A(1, 3) \rightarrow \vec{OA} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\begin{pmatrix} 7 \\ -3 \end{pmatrix} = \vec{OB} - \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \vec{OB}$$

$$\vec{OB} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

$$B(8, 0)$$

Q: $C(5, 7)$ and $\vec{AC} = \begin{pmatrix} 3 \\ 10 \end{pmatrix}$.
Find coordinates of A .

$$C(5, 7) \rightarrow \vec{OC} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

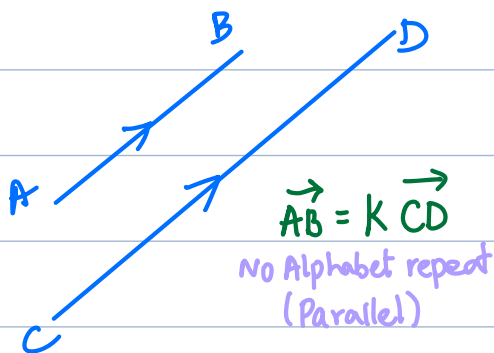
$$\vec{AC} = \vec{OC} - \vec{OA}$$
$$\begin{pmatrix} 3 \\ 10 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} - \vec{OA}$$

$$\vec{OA} + \begin{pmatrix} 3 \\ 10 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

$$\vec{OA} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 3 \\ 10 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$A(2, -3)$$

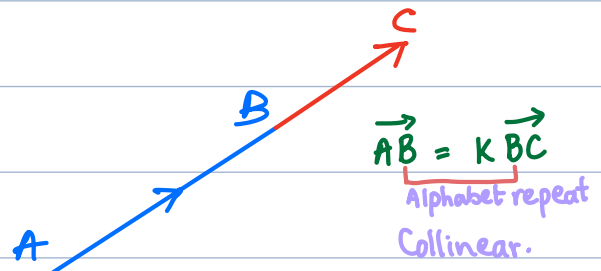
PARALLEL



$\nsubseteq$

COLLINEAR

(on the same line)



$$\underset{\substack{\text{First} \\ \text{Vector}}}{A} = \underset{\substack{\text{constant} \\ \text{(number)}}}{K} \underset{\substack{\text{Second} \\ \text{Vector}}}{B}$$

EXAM TIP

If the question already contains (K) , use another alphabet in our formula.

eg: $A = tB$, $A = xB$

Q: $\vec{AB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $\vec{CD} = \begin{pmatrix} 9 \\ k \end{pmatrix}$

Given that AB and CD are parallel, Find k.

$A = \cancel{k} B$

$A = t B$

$\begin{pmatrix} 9 \\ k \end{pmatrix} = t \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

$\begin{pmatrix} 9 \\ k \end{pmatrix} = \begin{pmatrix} 3t \\ 5t \end{pmatrix}$

$3t = 9$
 $t = 3$
 $k = 5t$
 $k = 5(3)$
 $k = 15$

Workings get shorter if we take vector with unknowns as our first vector.

Trigger words → Parallel
→ collinear (on the same line)

$\vec{AB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ $\vec{CD} = \begin{pmatrix} 9 \\ k \end{pmatrix}$

$\frac{3}{9} = \frac{5}{k}$

$3k = 45$
 $k = 15$

Shortcut working (Allowed in exams)

IF

PARALLEL,
 COLLINEAR
 ON THE SAME LINE

Q: $\vec{AB} = 7a + 12b$ $\vec{BD} = 21a + (t-1)b$

Given that A, B and D lie on same line.
Find t. (1 mark)

$$\frac{7a}{321a} = \frac{12b}{(t-1)b}$$

$$\frac{1}{3} = \frac{12}{t-1}$$

$$t-1 = 36$$

$$t = 37$$

Vectors:

- 1- Column Vectors + magnitude.
- 2- Equal vectors, Ratio method, Traffic flow diagrams.
- 3- position vectors
- 4- Parallel/collinear.

Notes
Revise

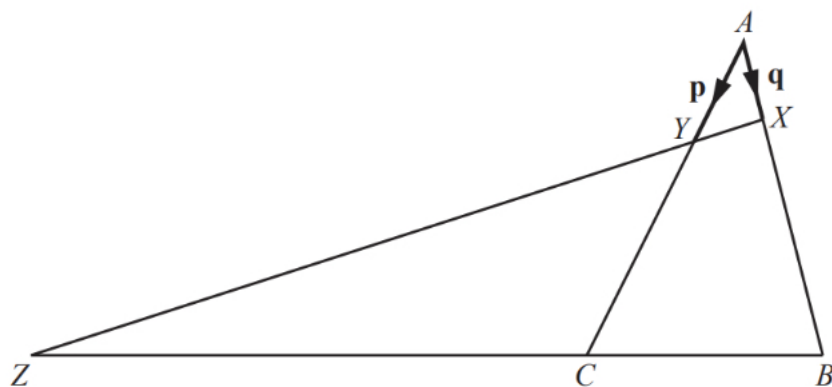
+

10 Questions
with help
from video

+

10-15 Q
Test
yourself.

Questions regarding area ratios in triangles with vectors is related to SIMILARITY.



In the diagram,

X is the point on AB where $AX = \frac{1}{4}AB$,

Y is the point on AC where $AY = \frac{1}{3}AC$,

Z is the point on BC produced where $CZ = 2BC$.

$\overrightarrow{AY} = \mathbf{p}$ and $\overrightarrow{AX} = \mathbf{q}$.

(a) Express, as simply as possible, in terms of $\mathbf{p}$ and $\mathbf{q}$,

(i) $\overrightarrow{XY}$,

Answer $\overrightarrow{XY} = \dots\dots\dots$ [1]

(ii) $\overrightarrow{BC}$,

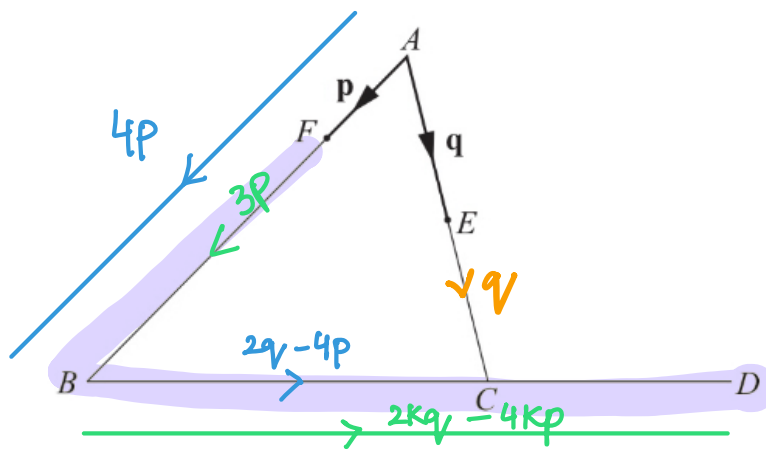
Answer $\overrightarrow{BC} = \dots\dots\dots$ [1]

(iii) $\overrightarrow{XZ}$.

Answer $\overrightarrow{XZ} = \dots\dots\dots$ [2]

(b) Hence find $XY : YZ$.

Answer $\dots\dots\dots : \dots\dots\dots$ [1]



In the diagram, F is the point on AB where $AF = \frac{1}{4} AB$.

E is the midpoint of AC .

$\vec{AF} = \mathbf{p}$ and $\vec{AE} = \mathbf{q}$.

$$\mathbf{p} = \frac{1}{4} \vec{AB}$$

$$\vec{AB} = 4\mathbf{p}$$

(a) Express, in terms of $\mathbf{p}$ and $\mathbf{q}$,

(i) $\vec{FE} = -\mathbf{p} + \mathbf{q} = \mathbf{q} - \mathbf{p}$

Answer $\mathbf{q} - \mathbf{p}$ [1]

(ii) $\vec{BC} = -4\mathbf{p} + 2\mathbf{q} = 2\mathbf{q} - 4\mathbf{p}$

Answer $2\mathbf{q} - 4\mathbf{p}$ [1]

(b) D is the point on BC produced such that $BD = kBC$.

(i) Express $\vec{FD}$ in terms of k , $\mathbf{p}$ and $\mathbf{q}$.

$$\vec{BD} = k(2\mathbf{q} - 4\mathbf{p})$$

$$\vec{BD} = 2k\mathbf{q} - 4k\mathbf{p}$$

$$\vec{FD} = 3\mathbf{p} + 2k\mathbf{q} - 4k\mathbf{p}$$

$$= 3\mathbf{p} - 4k\mathbf{p} + 2k\mathbf{q}$$

$$\vec{FD} = (3 - 4k)\mathbf{p} + 2k\mathbf{q}$$

Answer $(3 - 4k)\mathbf{p} + 2k\mathbf{q}$ [1]

(ii) Given that F , E and D are collinear, find the value of k .

$$\vec{FE} = \mathbf{q} - \mathbf{p}$$

$$\vec{FD} = 2k\mathbf{q} + (3 - 4k)\mathbf{p}$$

$$\frac{2k\mathbf{q}}{2k\mathbf{q}} = \frac{-(\mathbf{p})}{(3 - 4k)\mathbf{p}}$$

Answer $k = \frac{3}{2}$ [2]

$$\frac{1}{2k} = \frac{-1}{3-4k}$$

$$-2k = 3-4k$$

$$-2k + 4k = 3$$

$$2k = 3$$

$$k = \frac{3}{2}$$